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Report 149-M4

INVESTIGATION OF INHERENT STABILITY OF COMBUSTION SYSTEMS

Contract NAS 8-11016
Fourth Monthly Progress Report
For Period 1 October Through 31 October 1963

Prepared For
George C. Marshall Space Flight Center
Huntsville, Alabama
Attention: M-P&C-MEA

Prepared By
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OTS PRICE

XEROX $ 1.60ph

MICROFILM $

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OVERALL PROGRESS

A preliminary study of the transport properties of gases is reported in Appendix A. It is shown that all terms in Equation 11 of the first quarterly report[1] which contain derivatives of the velocity greater than first degree and first order are negligible due to the magnitude of the Reynolds number (10^7-10^8).

The major problem in applying the analyses of the unperturbed system[2] is the evaluation of the velocity derivative from experimental data in Equation 11. The velocity of the gases can be determined from the first derivative of the distance-time (x-t) curve obtained by streak photography with an accuracy of ±2.5%. However, the first derivative of the velocity (second derivative of the x-t curve) cannot be obtained with any degree of accuracy. This problem was overcome by writing Equation 11 in terms of measured pressure by use of Equations 4, 5a, and 10[3], as shown in Appendix B.

Equation 28a[4], which relates the reaction rate at a cross section due to the introduction of a pressure perturbation is converted to an axial distribution function in Appendix C.

Work was begun in relating the analyses in the quarterly report[5] to the conceptual model of the proposal.[6] This work is still in progress.

PROBLEMS ENCOUNTERED

No problems were encountered during this report period.

ERRATA TO FIRST QUARTERLY REPORT

An errata sheet specifying corrections to the first quarterly report is submitted with this report.

FUTURE WORK

It is expected that the work in relating the analyses in the quarterly report to the conceptual model will be completed during the next reporting period.

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1. Gaede, A.E., and Pickford, R.S., "Investigation of Inherent Stability of Combustion Systems," Quarterly Progress Report, NAS 8-11016, Oct 1963, p A-6.
2. Ibid., pp A-2 - A-8.
3. Ibid.
4. Ibid., p A-11.
5. Ibid., pp A-1 - A-11.
6. Astropower Proposal A63025 (Apr 1963).

FUNDS AND MAN-HOURS EXPENDED (To 3 November 1963)

Task 1

Man-hours expended	
This report period	619
Total	1016
Funds expended	
This report period	10,910
Total	19,080
Funds remaining, Task 1	$31,319.70

APPENDIX A

QUANTITATIVE EVALUATION OF THE ANALYSIS OF THE UNPERTURBED SYSTEM[1]

Equations 6a, 11, and 17 in the first quarterly report were derived in order to quantitatively describe the distribution functions of energy and rates of energy released due to chemical reactions during steady state combustion. These equations are

$$\frac{\hat{h}+q}{u_L^2} = \frac{\gamma}{\gamma-1}\left\{\frac{u}{u_L}\left(1-\frac{\gamma+1}{2\gamma}\frac{u}{u_L}\right)-\frac{1}{RePr}\left[\left\{\left[1-2\left(1+\frac{2}{3\gamma}Pr\right)\frac{u}{u_L}\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+\frac{4}{3Re}\frac{d}{d\xi}(u/u_L)\right]\right\}\frac{d}{d\xi}(u/u_L)+\frac{4}{3Re}\frac{u}{u_L}\frac{d^2}{d\xi^2}(u/u_L)\right]\right\} \tag{6a}$$

$$\frac{mu_L^2}{\kappa}\frac{d}{d\xi}\left(\frac{\hat{h}+q}{u_L^2}\right) = \frac{\gamma}{\gamma-1}\frac{mu_L^2}{\kappa}\left\{\left(1-\frac{\gamma+1}{\gamma}\frac{u}{u_L}\right)\frac{d}{d\xi}(u/u_L)\right.$$

$$+\frac{1}{RePr}\left\{\left[1-2\left(1+\frac{2}{3\gamma}Pr\right)\frac{u}{u_L}+\frac{4}{Re}\frac{d}{d\xi}(u/u_L)\right]\frac{d^2}{d\xi^2}(u/u_L)\right.$$

$$\left.\left.-2\left(1+\frac{2}{3\gamma}Pr\right)\left[\frac{d}{d\xi}(u/u_L)\right]^2+\frac{4}{3Re}\frac{u}{u_L}\frac{d^3}{d\xi^3}(u/u_L)\right\}\right\} \tag{11}$$

and

$$\frac{mu_L^2}{\kappa}\frac{d}{d\xi}\left(\frac{\hat{h}+q_d}{u_L^2}\right) = \frac{mu_L^2}{\kappa}\frac{d}{d\xi}\left(\frac{\hat{h}+q}{u_L^2}\right)-\frac{1}{ReSc}\frac{mu_L^2}{\kappa}\frac{d^2}{d\xi^2}\left(\frac{\hat{h}+q}{u_L^2}\right)$$

$$=\frac{mu_L^2}{\kappa}\frac{d}{d\xi}\left(\frac{\hat{h}+q}{u_L^2}\right)-\frac{Le}{RePr}\frac{mu_L^2}{\kappa}\frac{d^2}{d\xi^2}\left(\frac{\hat{h}+q}{u_L^2}\right) \tag{17}$$

where

$$Le = \frac{Pr}{Sc} \text{ (Lewis number)}$$

1. Gaede, A.E., and Pickford, R.S., "Investigation of Inherent Stability of Combustion Systems," First Quarterly Progress Report, Contract NAS 8-11016, (Oct 1963), pp A-4, A-6, A-7.

Equation 6a relates the total energy release per unit mass, q, at any position, ξ, in the combustor. Equation 11 yields the reaction rate of the propellants due to convection in units of energy per unit time per unit volume, and Equation 17 gives the reaction rate due to both convection and diffusion. All of the equations are in terms of known quantities associated with reactant and product properties, physical design of the combustor, and measurable parameters.

Relative magnitudes of the terms in these three equations have been calculated for a hypothetical combustor in an effort to determine the contributions of viscosity, heat conduction, and molecular diffusion to the distribution of energy release and reaction rates. The characteristics of this combustion system are:

Propellants:	Hydrogen and oxygen
Mixture ratio:	5:1
Initial pressure:	1000 psia
Combustion flame temperature:	$3309^{\circ}K$
Characteristic combustor length, $\mathcal{L}$:	20 in.

Assumptions:

a. Thermally choked, constant area, steady state flow

b. Gas properties and transport coefficients are mean values over the region of integration of the equations of motion.

c. The mean velocity is defined as the mean mass velocity.

d. The velocity gradient transverse to the axis is negligible.

e. The mixtures of reactants and products are thermally and calorically perfect.

Initial concern is with the magnitudes of the dimensionless parameters Re, Pr, and Le. These quantities involve the transport coefficients - viscosity (μ), thermal conductivity (K), and molecular diffusion of reactants to products ($\mathcal{D}$) - and the thermodynamic properties-specific heat (c_p) and density (ρ). In addition, the Reynolds number requires knowledge of either velocity (u) or flow rate per unit area (m) and the characteristic combustor length ($\mathcal{L}$). The characteristic combustor length, $\mathcal{L}$, is specified as 20 in. This number is based on the similarity between $\mathcal{L}$ and characteristic length, L^*, associated with rocket combustion chamber design. $\mathcal{L}$ in this case represents only the portion of L^* which is required for combustion because the hypothetical test model is assumed to operate on premixed gaseous propellants. The flow rate per unit area used in the Reynolds number is calculated from data of Gaede[2] by assuming a Mach number of unity at the combustor exit.

The large flow rates per unit area and the combustion cause highly turbulent flow in the combustor. Because transport properties are derived from molecular theory, there may be doubt as to their applicability in highly turbulent flow. From Bartz'[3] studies of turbulent flow in reaction engine combustion chambers, it appears that the major resistance to transport occurs in the wall surface boundary layer where molecular transport is known to dominate. This explains the success in correlating momentum, energy, and mass transport in turbulent flow by using similarity parameters containing the molecular transport properties. Similarity parameters based on the molecular transport properties have also been used successfully in semi-theoretical treatment of turbulent boundary layers.

The terms in Equations 6a, 11, and 17 have been evaluated for two axial positions in the combustor: the first for the mixture preceding initiation of combustion, where the major concern was with physical properties and velocities of reactants; and the second at completion of combustion, where knowledge of physical properties and velocities of products was required. The following composition of reactant and product mixtures was used in the evaluation:

Reactants	Mole fr, x_i	Wt fr, m_i
H_2	0.760	0.167
O_2	0.240	0.833

Products	Mole fr, x_i	Wt fr, m_i
H_2	0.375	0.063
H_2O	0.625	0.937

2. Gaede, A.E., and Pickford, R.S., "Investigation of Inherent Stability of Combustion Systems," Second Monthly Progress Report, Contract NAS 8-11016 (Sept 1963).
3. Bartz, D.R., "The Role of Transport Properties in the Problems of Jet Engines and Rockets," from Cambel and Fenn, Transport Properties in Gases (1958).

Component viscosities, thermal conductivities, and specific heats were obtained from the work of Svehla.[4] Brokaw's[5] alignment charts were utilized to compute the transport properties of the mixtures. It was assumed that frozen conductivities and specific heats were sufficiently accurate for the present evaluation of the dimensionless quantities. Corrections to the transport properties for the higher pressures were found unnecessary in all cases because components were well above critical temperatures and/or at relatively low reduced pressures, p/p_{crit}, during combustion and expansion in the chamber.[6] Table I lists property data and resulting dimensionless parameters for the two sets of conditions in the combustion.

TABLE I

PROPERTY DATA AND DIMENSIONLESS PARAMETERS

	Comp	μ	k	c_p	Re	Pr	Le
Condition 1 Reactants		lb/sec ft	BTU/sec ft °F	BTU/lb °F			
P = 528 psi	H_2	6.0×10^{-6}	28.9×10^{-6}	3.5	-	-	-
T = 300°K	O_2	13.8×10^{-6}	4.42×10^{-6}	0.290	-	-	-
	Total mixture	11.2×10^{-6}	19.1×10^{-6}	0.826	80.8×10^{6}	0.485	-
Condition 2 Products at completion of combustion							
P ~ 250 psi	H_2	24.1×10^{-6}	145×10^{-6}	4.24	-	-	-
T = 2500°K	H_2O	53.2×10^{-6}	52.9×10^{-6}	0.718	-	-	-
	Total mixture	50.1×10^{-6}	99.4×10^{-6}	0.939	18.0×10^{6}	0.473	1.7

4. Svehla, R.A., "Estimated Viscosities and Thermal Conductivities of Gases at High Temperatures," NASA TR R-132 (1962).
5. Brokaw, R.S., "Alignment Charts for Transport Properties, Viscosity, Thermal Conductivity, and Diffusion Coefficients for Nonpolar Gases and Gas Mixtures at Low Density," NASA TR R-81 (1961).
6. McAdams, W., Heat Transmission, Third Edition, 1954, pp 459, 468.

The Lewis number was obtained directly from Brokaw,[7] who presents data on this parameter for combustion products of a hydrogen-oxygen propellant combination operating near maximum specific impulse at temperatures very close to those specified in Table I.

To complete evaluation of the terms of Equations 6a, 11, and 17, typical ratios of velocities (u/u_L) and first, second, and third derivatives of the velocities with respect to axial position, ξ, were obtained from experimental data taken in a two-dimensional RP1-LOX combustor operated at 300 psig.[8] The highest axial velocity gradient, du/dx, shown in the test data is 6240/sec. Nondimensionalized by L^* and u_L, this reduces to $d(u/u_L)/d\xi = 2.29$. For the hypothetical H_2-O_2 combustor, the maximum $d(u/u_L)/d\xi$ should be less than 2.29 because the combustion occurs over a greater length. However, for the purpose of conservative evaluation of the relative magnitudes of terms in Equations 6a, 11, and 17, a maximum value of $d(u/u_L)/d\xi = 100$ was assumed. It is also assumed that the axial velocity profile in the hypothetical combustor is of the form

$$\frac{u}{u_L} = C_1 - C_2 e^{-C_3 \xi}$$

The constants C_1, C_2, and C_3 are evaluated from the conditions that $d(u/u_L)/d\xi = 100$ and $u/u_L = 0.1$ at $\xi = 0$, and $u/u_L = 0.5$ at $\xi = 1$. The following equations result:

$$\frac{u}{u_L} = 0.400\,(1.25 - e^{-250\,\xi}) \tag{1}$$

$$\frac{d}{d\xi}(u/u_L) = 100\,e^{-250\,\xi} \tag{2}$$

$$= 100 \;@\; \xi = 0$$

$$\rightarrow \quad 0 \;@\; \xi = 1.0$$

7. Brokaw, R.S., "The Lewis Number," Second Symposium on Thermophysical Properties, Princeton University (1962).
8. Lambiris, S., and Combs, L.P., "Steady-State Combustion Measurements in a LOX/RP1 Rocket Chamber," Combustion Institute Paper WSS/CI 61-13 (1961).

$$\frac{d^2}{d\xi^2}(u/u_L) = -25{,}000\, e^{-250\,\xi} \tag{3}$$

$$= -25{,}000 \ @ \ \xi = 0$$

$$\rightarrow \quad 0 \ @ \ \xi = 1.0$$

$$\frac{d^3}{d\xi^3}(u/u_L) = 6.25 \times 10^6\, e^{-250\,\xi} \tag{4}$$

$$= 6.25 \times 10^6 \ @ \ \xi = 0$$

$$\rightarrow 0 \ @ \ \xi = 1.0$$

The equations were rewritten in the following form to facilitate relative comparison of each term.

$$\frac{\hat{h}+q}{u_L^2} = \left(\frac{\gamma}{\gamma-1}\right)\frac{u}{u_L}\left(1 - \frac{\gamma+1}{2\gamma}\frac{u}{u_L}\right) - \left(\frac{\gamma}{\gamma-1}\right)\frac{1}{RePr}\frac{d}{d\xi}(u/u_L)$$

$$+ \left(\frac{\gamma}{\gamma-1}\right)\frac{2}{RePr}\left(1 + \frac{2}{3\gamma}Pr\right)\frac{u}{u_L}\frac{d}{d\xi}(u/u_L) - \left(\frac{\gamma}{\gamma-1}\right)\frac{4}{3Re^2Pr}\left(\frac{d}{d\xi}(u/u_L)\right)^2$$

$$- \left(\frac{\gamma}{\gamma-1}\right)\frac{4}{3Re^2Pr}\frac{u}{u_L}\frac{d^2}{d\xi^2}(u/u_L) \tag{6a}$$

$$\frac{d}{d\xi}\left(\frac{\hat{h}+q}{u_L^2}\right) = \left(\frac{\gamma}{\gamma-1}\right)\left(1 - \frac{\gamma+1}{\gamma}\frac{u}{u_L}\right)\frac{d}{d\xi}(u/u_L) + \left(\frac{\gamma}{\gamma-1}\right)\frac{1}{RePr}\frac{d^2}{d\xi^2}(u/u_L)$$

$$- \left(\frac{\gamma}{\gamma-1}\right)\frac{2}{RePr}\left(1 + \frac{2}{3\gamma}Pr\right)\frac{u}{u_L}\frac{d^2}{d\xi^2}(u/u_L)$$

$$+ \left(\frac{\gamma}{\gamma-1}\right)\frac{4}{Re^2Pr}\frac{d}{d\xi}(u/u_L)\frac{d^2}{d\xi^2}(u/u_L) - \left(\frac{\gamma}{\gamma-1}\right)\frac{2}{RePr}\left(1 + \frac{2}{3\gamma}Pr\right)\left(\frac{d}{d\xi}(u/u_L)\right)^2$$

$$+ \left(\frac{\gamma}{\gamma-1}\right)\frac{4}{3Re^2Pr}\frac{u}{u_L}\frac{d^3}{d\xi^3}(u/u_L) \tag{11}$$

$$\frac{d}{d\xi}\left(\frac{\hat{h}+q_d}{u_L^2}\right) = \frac{d}{d\xi}\left(\frac{\hat{h}+q}{u_L^2}\right) - \frac{Le}{RePr}\frac{d^2}{d\xi^2}\left(\frac{\hat{h}+q}{u_L^2}\right) \tag{17}$$

The terms of Equations 6a, 11, and 17 were evaluated for $\gamma = 1.2$ utilizing Equations 1, 2, 3, and 4 and the similarity parameters in Table I. The comparison of terms in Table II was made at the locations $\xi = 0$ and $\xi = 1.0$.

It was concluded that terms of the equations which contain the similarity parameters 1/Re are sufficiently small to be omitted. The equations may be simplified and written:

$$\frac{\hat{h}+q}{u_L^2} = \frac{\gamma}{\gamma-1}\left[\frac{u}{u_L}\left(1 - \frac{\gamma+1}{2\gamma}\frac{u}{u_L}\right)\right] \tag{6a}$$

$$\frac{mu_L^2}{\mathcal{K}}\frac{d}{d\xi}\left(\frac{\hat{h}+q}{u_L^2}\right) = \frac{\gamma}{\gamma-1}\frac{mu_L^2}{\mathcal{K}}\left[1 - \frac{\gamma+1}{\gamma}\frac{u}{u_L}\right]\frac{d}{d\xi}(u/u_L) \tag{11}$$

for

$$0 \leq \xi \leq 1$$

Interpreted physically, the results show that viscosity, heat conduction and molecular diffusion in the axial direction contribute only slightly to the energy release and reaction rate distributions in the combustor during steady state operation.

TABLE II

Comparison of Terms in Equations 6a, 11, and 17

Equation	Axial Position (ξ)	Term No.	Value
6a	0	1	0.545
	0	2	1.53×10^{-5}
	0	3	0.390×10^{-5}
	0	4	2.52×10^{-11}
	0	5	-0.631×10^{-11}
6a	1	1	0.252
	1	2	0
	1	3	0
	1	4	0
	1	5	0
11	0	1	490
	0	2	-3.85×10^{-3}
	0	3	-9.75×10^{-4}
	0	4	-1.89×10^{-8}
	0	5	3.88×10^{-3}
	0	6	1.58×10^{-9}
11	1	1	0
	1	2	0
	1	3	0
	1	4	0
	1	5	0
	1	6	0
17	0	1	490
	0	2	$\sim 10^{-3}$
17	1	1	0
	1	2	0

APPENDIX B

CHANGE OF DEPENDENT VARIABLE IN EQUATION 11[1]

The large errors encountered in measuring velocity derivations from streak photographs has made it advisable to change from the dimensionless velocity (u/u_L) to the dimensionless pressure $(p/m\,u_L)$ in Equation 11, which at present relates the axial distribution of the reaction rate to u/u_L. Multiplying the left side of Equation 5a[2] by ρu, dividing by m, rearranging, and differentiating yields

$$\frac{u}{u_L} = 1 - \frac{p}{mu_L} \tag{5}$$

and

$$\frac{d}{d\xi}(u/u_L) = -\frac{d}{d\xi}(p/m\,u_L) \tag{6}$$

Substitution of Equations 5 and 6 into Equation 11[3] yields the reaction rate in terms of the dimensionless pressure

$$\frac{mu_L^2}{\mathcal{X}}\frac{d}{d\xi}\left(\frac{h+q}{u_L^2}\right) = \frac{1}{\gamma-1}\frac{mu_L^2}{\mathcal{X}}\left[1-(\gamma+1)\frac{p}{mu_L}\right]\frac{d}{d\xi}(p/mu_L) \tag{11a}$$

in which the terms multiplied by $1/Re$ and $1/Re^2$ have been omitted (see Appendix A).

1. Gaede, A.E., and Pickford, R.S., "Investigation of Inherent Stability of Combustion Systems," First Quarterly Progress Report, Contract NAS 8-11016 (Oct 1963), p A-6.
2. Ibid., p A-4.
3. Ibid., p A-6.

APPENDIX C

CONVERSION OF EQUATION 28[1] TO AN AXIAL DISTRIBUTION FUNCTION

The total energy released per unit time by a shock wave over a cross section of the flow is equal to $m'q'A'$. Dividing the total energy released by $A\varkappa\Delta\xi$ yields

$$\frac{m'A'q'}{A\varkappa\Delta\xi} = \frac{\gamma}{\gamma-1}\,\frac{m'v_L^2A'}{A\varkappa\Delta\xi}\left[\frac{v_2}{v_L}\left(1-\frac{\gamma+1}{2\gamma}\,\frac{v_2}{v_L}\right)-\frac{v_1}{v_L}\left(1-\frac{\gamma+1}{2\gamma}\,\frac{v_1}{v_L}\right)\right] \qquad (28a)$$

in units of energy per unit time per unit volume.

1. Gaede, A.E., and Pickford, R.S., "Investigation of Inherent Stability of Combustion Systems," First Quarterly Progress Report, Contract NAS 8-11016 (Oct 1963), p A-11.

ERRATA TO REPORT 149-Q1

Page 2, Line 17: close parentheses following ". . . heat release rate)."

Page A-1, Line 3: change "steady stage" to "steady state."

Page A-4, Line 14: change "independent" to "dependent."
Line 15: change "dependent" to "independent."

Page A-5, Line 9: change "independent" to "dependent."
Line 11: change "independent" to "dependent."

Page A-8, Line 12: change "$q = q_d$" to "$q = q_o$."

Page A-9, Line 11: change "$\mathcal{L}$" to "unity."

Page A-10, Line 11: change "independent" to "dependent."
Lines 11 and 12: change "dependent" to "independent."
Equation 25a, change

$$\frac{h+q'}{v_L^2}$$

to

$$\frac{\overline{h}+q'}{v_L^2}$$

Page A-11, Equation 28: replace w, the transverse dimension of the combustor, with ϵ, the shock thickness.